

liquid line to the pump. A measured volume of liquid is introduced into the still with a hypodermic syringe through one of the sample ports. Minute samples of the equilibrium vapor condensate and equilibrium liquid are removed from the apparatus without disturbing the pressure on the system at pressures below or above atmospheric pressure by a pressure-lok liquid microsyringe.\* The composition of the samples are determined by chromatographic analysis or by a measurement of the refractive

\* Available only from Precision Sampling Corp., Baton Rouge, Louisiana.

index, having previously established the relation between the refractive index and composition experimentally.

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## The Influence of Inlet Flow Disturbances on Transition of Poiseuille Pipe Flow

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The stability of fully developed Poiseuille pipe flow to infinitesimal disturbances has been abundantly demonstrated theoretically. Recent work (Huang and Chen, 1974a, b; Sarpkaya, 1975; Wygnanski and Champagne, 1973; Wygnanski et al., 1975) with developing flows and inlet flows has suggested rather strongly that the source of instabilities which lead to the well-known transition to turbulence in a Poiseuille pipe flow lies in the entry of the pipe.

As a part of a study of transitional flow phenomena in concentric annuli, the experiments reported in this note were performed. A series of large disturbances were introduced into a fully developed laminar flow which was impinging upon the entry of a second pipe. The effects of these disturbances on the transitional flow behavior of the downstream flow were observed and are reported.

#### EXPERIMENTAL

A recirculating flow loop, described in detail elsewhere (Peterson, 1979), was used for the measurements herein reported. Solutions of Dow polyglycol\* 15-200 in water were pumped through a 7.3 m long vertical test section consisting of a 3.66 m long, 38.1 mm ID heavy walled aluminum pipe in series with a 3.66 m long, 19.1 mm ID aluminum pipe. A precision machined aluminum block, illustrated in Figure 1, coupled the pipes and served as the disturbance-generator holder.

Flow rates were measured by calibrated turbine flowmeters, and pressure losses were measured in the smaller diameter pipe with sensitive differential pressure transducers. Data were collected in the smaller diameter pipe for each of the following configurations:

1. Disturbances caused only by the 2:1 diameter reduction.
2. Symmetric disturbance device (Figure 2a) pinned rigidly in the holder.
3. Swirl disturbance device (Figure 2b) in the holder allowed to rotate freely about the pipe axis as fluid passed

\*Dow Chemical Co. trademark.

through, thus imparting a swirl to the disturbances generated.

From the pressure drops and flow rates obtained, friction factors ( $f = D\Delta p / 2L\rho V^2$ ) and Reynolds numbers ( $Re = DV\rho/\mu$ ) were calculated. For Poiseuille pipe flow,

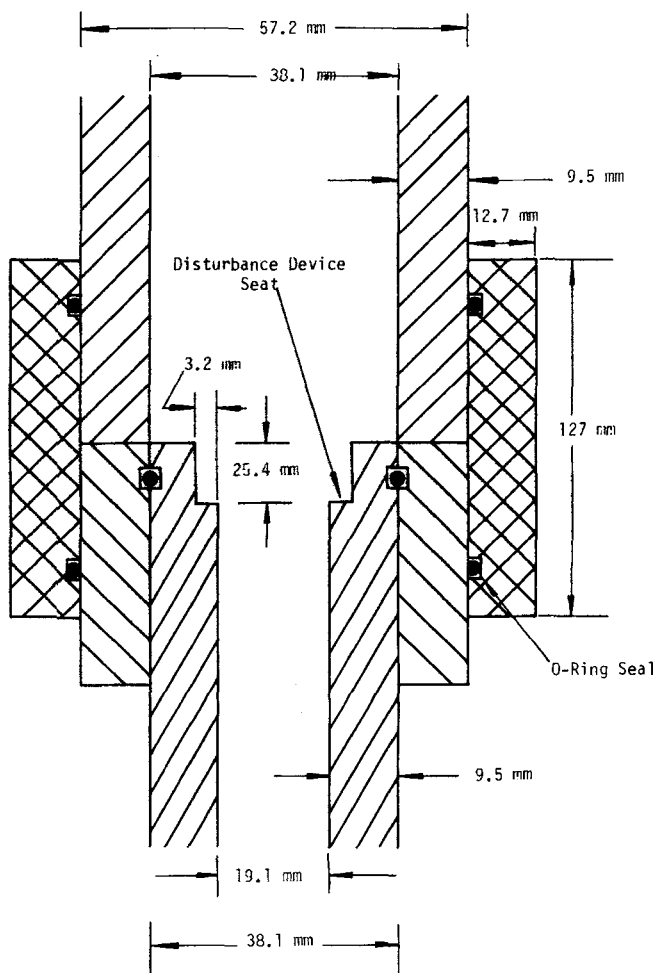


Fig. 1. Details of pipe coupling and disturbance device holder.

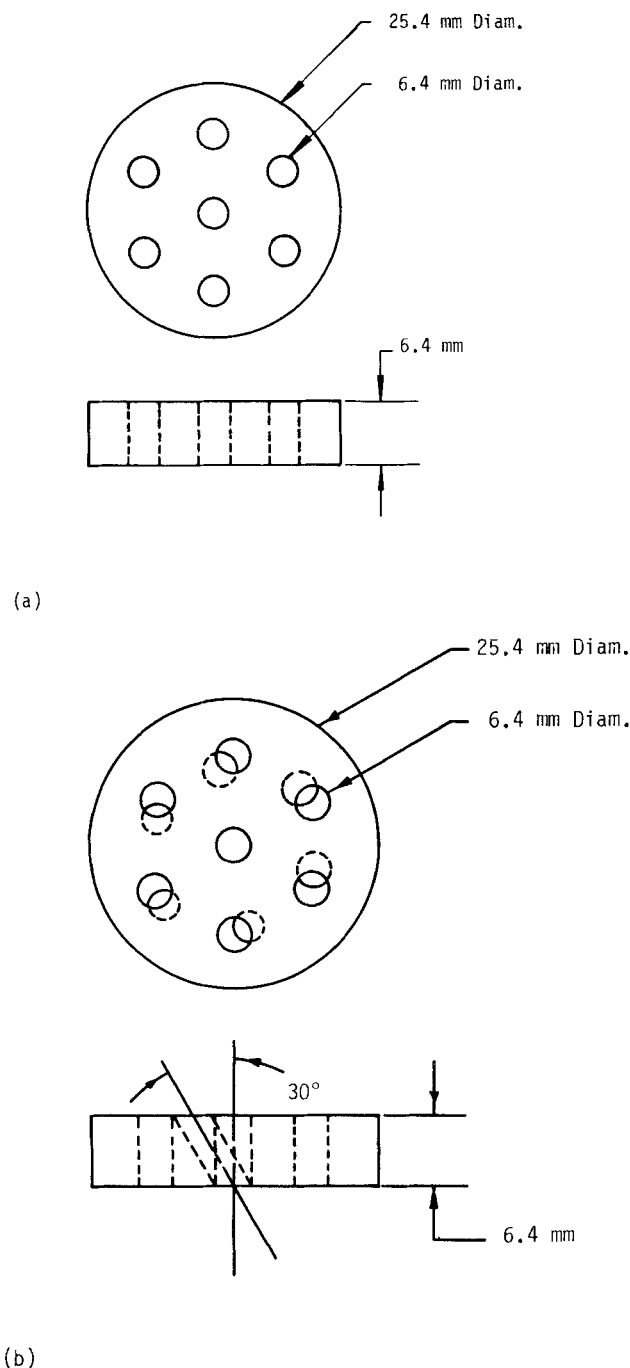


Fig. 2. (a) Details of symmetric disturbance device. (b) Details of swirl disturbance device. Material for both devices is aluminum.

the solution of the equation of motion may be expressed as  $fRe = 16$ , suggesting that a plot of the product  $fRe$  as a function of  $Re$  should readily reflect any deviations from acceptable behavior. The data obtained are so plotted in Figure 3, with  $Re$  being based on the 19.1 mm diameter pipe. Because of the diameter difference of the two pipes,  $Re_2 = 2Re_1$  ( $Re_2$  is small pipe,  $Re_1$  is large pipe). For the data in Figure 3,  $fRe = 16.04 \pm 0.17$  at the 95% confidence interval.

The coupling point is 96 diam from the entry plane of the larger pipe. Brodkey (1967) estimates  $L/D = 0.05$   $Re$  for fully developed Poiseuille flow in a pipe. This suggests that the velocity profile impinging upon the coupling joint should be the parabolic for  $Re_1 = 1920$ , ( $Re_2 = 3840$ ). It was not possible to measure the velocity profile directly.

## INTERPRETATION

For configuration 1 (open circle data in Figure 3), it is apparent that disturbances resulting from a 2:1 sharp edged diameter decrease have no effect on the downstream fully developed Poiseuille flow until  $Re_2 = 3920$  ( $Re_1 = 1960$ ). This is just slightly greater than the maximum value of  $Re_1$  for which the velocity profile arriving at the coupling is expected to cease being fully developed Poiseuille (Brodkey, 1967). Thus, it appears that only when the impinging velocity profile begins to differ significantly from the Poiseuille parabola does the moderately large but symmetric disturbance due to the 2:1 diameter decrease cause transition in the flow.

The data points for configuration 2 (triangles in Figure 3) suggest the existence of an amplitude effect. Here the disturbances introduced into the impinging parabolic flow are symmetrical but periodic about the pipe axis, and of much larger amplitude than in configuration 1. These disturbances resulted in  $Re_{c2} = 2750$ , ( $Re_1 = 1375$ ). Apparently, therefore, if the disturbance is large enough, even fully developed Poiseuille flow will break down. Because of the complex nature of the disturbances generated, no quantitative measure of large enough can be deduced from these data. All that can be said is that qualitatively they must be quite large.

The solid data points in Figure 3 correspond to configuration 3, where a swirling three-dimensional disturbance of unknown but large magnitude was introduced into the flow. This device caused  $Re_{c2} = 1800$  ( $Re_1 = 900$ ), which clearly suggests that the impinging Poiseuille flow is unstable to large amplitude swirling types of disturbances.

The results of this study tend to confirm the observations of Bhat (1966), Fox et al. (1968), Sarpkaya (1975), and Wygnanski and Champagne (1973). We conclude that fully developed Poiseuille flow is stable to low amplitude symmetric disturbances, but that high amplitude symmetric disturbances and high amplitude swirling disturbances can induce transition. There appears to exist a threshold amplitude (apparently quite large) below which disturbances do not cause transition. We were unable to determine quantitatively its magnitude. Also, developing pipe flows, having velocity profiles significantly different from the Poiseuille parabola, appear

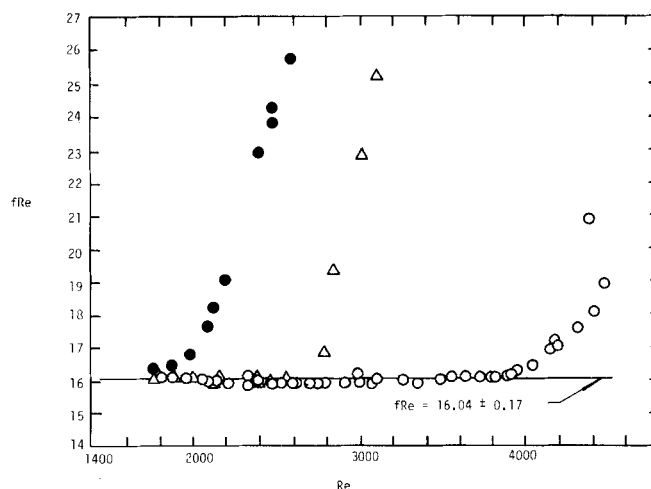


Fig. 3. Experimental data. Open circles correspond to no disturbance device in place. Triangles correspond to symmetric disturbance device in place. Solid circles correspond to swirl disturbance device in place. Reynolds numbers and friction factors are based on the diameter of the small pipe. Variance on the laminar value for  $fRe$  represents 95% confidence interval.

to be unstable to smaller amplitude disturbances. Further work is needed to identify quantitative limits.

#### ACKNOWLEDGMENT

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## The Influence of Lateral Pressure Variations on the Stability of Rapidly Evaporating Liquids at Reduced Pressure

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As the evaporation rate of a liquid under vacuum ( $<1$  torr) is gradually increased, a critical point is reached at which the gas-liquid interface spontaneously transforms from a relatively quiescent surface to one which is intensely turbulent, producing a marked increase in the evaporative flux (Hickman, 1952, 1972). The instability is attributed to the differential vapor-recoil mechanism in which the interface is unstable to local variations in evaporation rate which are produced by local perturbations in surface temperature, local surface depressions being produced by the recoil force exerted on the surface by the rapidly departing vapor and sustained liquid flows being driven by the resultant shear exerted on the liquid surface by the vapor (Palmer, 1976). Interfacial contamination can play a major role in determining the stability of the liquid surface. In particular, both linear stability analysis and experiments have shown that the addition of trace amounts of a nonvolatile surfactant to a liquid of moderate surface tension can dramatically increase the stability limit for vapor-recoil instability (Palmer, 1977).

In addition to establishing criteria for vapor-recoil instability, the linear stability analysis of a rapidly evaporating liquid also revealed that the interface may become unstable owing to lateral variations in gas phase pressure at the interface which are produced when fluid streamlines are distorted near the deforming interface. Called the fluid-inertia mechanism, this mode of instability is unaffected by the degree of surfactant contamination present in the system and therefore establishes an upper bound on the stability of the interface.

Although the analysis of Palmer (1976) accounts for the effect of fluid inertia on pressure fluctuations through the equation of momentum conservation for the gas

phase, it does not consider how these lateral pressure variations influence the local evaporation rate and thus the criteria for instability. The rate of evaporation depends not only on surface temperature but also on gas phase pressure (see Schrage, 1953). In the vicinity of surface depressions, inertial effects produce an increase in gas phase pressure and consequently a decrease in the local evaporation rate. From the viewpoint of vapor-recoil instability, the effect is to moderate the local increase in evaporation rate and recoil force which accompanies local increases in surface temperature. From the viewpoint of fluid-inertia instability (for which the surface temperature is presumed constant), the effect of local pressure variations on evaporative flux is to produce a recoil force which is maximum on surface crests and minimum in depressions, thus resisting further deformation of the interface.

In summary, then, the effect of gas phase pressure on evaporation rate is to increase the stability of the system to disturbances amplified by both the differential vapor-recoil and fluid-inertia mechanisms. The purpose of the current paper is to assess the importance of this stabilizing influence with the aid of linear stability analysis.

#### MATHEMATICAL ANALYSIS

The linear stability analysis of a steadily evaporating liquid in which pressure effects on evaporation rate are considered differs from that presented by Palmer (1976) only in the expression for the perturbed evaporation rate  $\eta'$ , which for the present case is given by

$$\eta' = \left( \frac{\partial \eta}{\partial T} \right) T' + \left( \frac{\partial \eta}{\partial p_v} \right) p_v'$$

The result is that the term  $N_H(T_L - B)$  in Equations (24) and (27) of Palmer (1976) is replaced by  $N_H(T_L - B) - N_{p_v}$ . All the other differential equations and